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An analytical method of modeling the process zone near the tip of an interface crack due to its kinking from the interface of quasi-elastic materials

Anatoly Kaminsky [a,*], Mykhailo Dudyk [b], Yuliia Reshitnyk [b], Yuri Chornoivan [a]

[a] S.P. Timoshenko Institute of Mechanics, National Academy of Sciences, Nesterov st., 3, Kyiv, 03057, Ukraine
[b] Pavlo Tychyna Uman State Pedagogical University, Sadova st. 2, Uman, 20300, Ukraine

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ABSTRACT

In order to predict the kinking of an interface crack in a quasi-elastic material under the plane strain conditions, a mathematical model of the process zone is formulated. In this model, the process zone extends from the tip of the interface crack at an angle to the plane interface of two different homogeneous isotropic materials. In the process zone, the strength criterion of Mises-Hill type is fulfilled, which implies a quasi-brittle mechanism of fracture. The problem of determining the zone parameters is reduced to the Wiener–Hopf vector functional equation, for which an approximate analytical solution has been found. Equations are derived for calculating the length and orientation of the process zone, the phase angle of stresses, and the amount of energy dissipation in the zone, as well as the crack tip opening. Under biaxial load conditions, a numerical analysis of the process zone parameters dependence on the loading values and the characteristics of the bounded materials was carried out. It is found that the studied parameters are significantly influenced by the phase angle of the external load and the ratio of the tensile strength to the shear strength for the material in which the process zone develops. The analysis of experiments on the kinking of the interface crack showed the possibility of using the developed model in studying the fracture of some composites of quasi-elastic materials.

1. Introduction

In modern industry, piecewise homogeneous materials, e.g. composites, laminates, materials with sandwich geometry, adhesive and welded joints of various materials, etc., have become widely used. The scopes and variety of conditions of their use are quite numerous, from everyday life and medicine to aviation and rocket technology, which imposes increased requirements on the strength of the specified materials.

The problem of the strength of piecewise homogeneous bodies, which is key to the reliability of their operation, at the basic level of research can be reduced to solving the problems of the initiation and propagation of a crack in a bimaterial joint. The set of such problems includes the problems of propagation of an interface crack, crossing, sliding or branching of a crack extending to the interface. This set contains the problem of an interface crack, which is the main source of destruction of piecewise homogeneous bodies, and thus it is of great interest.

Studies of the interface crack within the framework of the classical open crack model, initiated by Williams (1959), England (1965), Erdogan (1965), Rice and Sih (1965) and others, revealed the physical incorrectness of the analytical solution of the corresponding boundary value problem of the theory of elasticity in the form of interpenetration of the crack faces and oscillating stresses when reaching the crack tip. The alternative model of the interface crack proposed by Comninou (1977), which assumed the contact between the faces near the crack tip, is able to eliminate the shortcomings of the classical model of the interface crack. However, for some load configurations of a piecewise homogeneous body with an interface crack (e.g., under the normal tension or near one of the internal crack tips under the shear loading), the Comninou model predicts an extremely small extent of the contact zone (Comninou, 1977; Comninou and Schmueser, 1979; Dundurs and Gautesen, 1988), which for physical reasons means the inexistence of contact and returns the problem to the classical model of an open interface crack.

To date, the solution to the interface crack problem can be seen in taking into account the process zones that emerge in the vicinity of the tips due to the presence of stress concentration. The results of the first analytical calculations of the process zone parameters in the bound material at the planar interface of isotropic (Kaminskii et al., 1995, 1999) and anisotropic (Loboda and Sheveleva, 2003) materials when

* Corresponding author.
E-mail addresses: fract@inmech.kiev.ua (A. Kaminsky), dudik_m@hotmail.com (M. Dudyk), dikhtiarenko_iu@udpu.edu.ua (Y. Reshitnyk), yurchor@ukr.net (Y. Chornoivan).

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modeling the zones with displacement discontinuity lines (this corresponds to the models of Barenblatt (1962), Dugdale (1960) and Leonov and Panasyuk (1959)) indicated the significant influence of the process zones on the stressed state near the tips of interface cracks. A more complex model of the cohesive zone (Needleman, 1990; Tvergaard and Hutchinson, 1992; Tvergaard, 2004; Jin and Sun, 2005), which is based on specific relations between the stress and the relative displacement of the process zone faces, has gained significant use for the numerical modeling of interfacial fracture processes in composites.

At the same time, mathematical difficulties delayed the study of new models of process zones in bounded materials. The relevance of such studies is due to their direct connection with the problem of the interface crack kinking (its deviation from the interface of materials). This problem is crucial in the design of gravity dams (Kishen, 2005), aerospace and automobile structures with fiber-reinforced composites (Xie et al., 2005; Katsiropoulos et al., 2012; Campilho et al., 2008), etc.

The theoretical foundations of interface crack kinking studies were laid by He and Hutchinson (1989), but their implementation within the framework of the open crack model ran into the problem of ambiguity in predicting kinking angles due to their dependence on the choice of the reference length, which is included in the description of the stressed state near crack tips (Geubelle and Knauss, 1994; Rice, 1988). This problem can be avoided when using the Comninou contact model, but it gives only estimated values of the interface crack kinking angle, which are quite inaccurate (Xie et al., 2005). Therefore, theoretical studies usually use a stricter scheme for solving the problem of interface crack kinking, which is based on the introduction of a secondary small-scale crack-cut, emerging from the tip of the initial interface crack in one of the materials of the composite joint. After finding the parameters of the stress–strain state near the tip of the secondary crack, its orientation is determined by some criterion, which indicates the direction of kinking of the interface crack. The most used and effective are the criteria of the maximum rate of energy release and local symmetry (the local coefficient of stress intensity near the tip of the secondary lateral crack $k_{II} = 0$), which generally lead to close numerical values of the kinking angle.

In particular, in the works of Leblond and Frelat (2001, 2004), using the finite element method and the principle of local symmetry, the kinking angles of the interface crack at the initial contact of its faces were calculated. In Shan et al. (2008), using the dislocation technique allows for reducing the problem of the interface crack kinking with contacting faces to a singular integral equation, from which the stress intensity factors at the tip of the kinked crack are determined by numerical integration and the crack kinking angles are estimated according to the criterion of the maximum energy release rate.

The analytical solution to the problem of interface crack kinking is obtained in works (Kaminsky et al., 2007, 2009) using the Wiener–Hopf method. The calculation scheme for the angle of the crack kinking is based on the introduction of a small-scale process zone spreading from the tip of the initial open crack in a more compliant material. The process zone is presented as a segment of a straight line of normal displacement discontinuity, on which the normal stress is equal to the tensile strength of the material (Barenblatt–Leonov–Panasyuk model). Formally, this scheme is similar to the scheme with a small-scale open lateral crack, differing from it by the constant adhesion forces between the crack faces and the lack of tangential displacement discontinuity.

The numerical values of the kinking angles of the interface crack, obtained in Leblond and Frelat (2001, 2004), Shan et al. (2008), Kaminsky et al. (2007, 2009) and other works within the framework of various calculation schemes and criteria for determining the angles, turn out to be close in magnitude and, in general, are in agreement with the experimental results in the case of composite bounding of elastic materials with a brittle fracture mechanism. However, the attempts to apply these schemes to quasi-brittle materials were less successful. The reason for this is the neglect of plastic properties of bounded materials that can significantly influence the direction of further crack propagation. To tackle this issue, this work proposes a model of the process zone, which is a generalization of the model considered in Kaminsky et al. (2007, 2009) with the requirement of the tangential displacement discontinuity. The condition for assigning the material to the process zone of destruction is the fulfillment of the quadratic strength criterion of Mises-Hill type, in which the parameters are the limiting values of the tangential and tensile stresses in the zone. The basis for using the Mises-Hill type criterion is its effectiveness, experimentally confirmed when building models of process zones in orthotropic homogeneous bodies (Kaminsky et al., 2011), as well as illustrated in some alternatives of the interface fracture theory within the framework of the cohesive zone model (Jung Lee et al., 2010; Campilho et al., 2008).

2. Problem statement

Under the plane strain conditions, the problem of determining the parameters of the process zone near one of the tips of an interface crack of length L, located on the planar boundary between two different homogeneous isotropic quasi-elastic materials, is considered. The quasi-elasticity here means that they are elastic everywhere but the process zone. Materials are characterized by Young's moduli E_1, E_2 and Poisson ratios ν_1, ν_2 respectively. Taking into account the experimentally confirmed effectiveness of the models developed in Kaminsky et al. (2011), the solution to the problem is based, with some specifics, on the initial conditions used in Kaminsky et al. (2011):

1. From the tip of the crack, as a stress concentrator, in the more compliant (the first for the sake of certainty) material of the investigated composite body at relatively low load levels, a narrow, small-scale process zone of length l ($l \ll L$) develops.
2. In accordance with the models of Barenblatt (1962), Dugdale (1960), Leonov and Panasyuk (1959) and some alternatives of the cohesive zone model (Campilho et al., 2008; Jung Lee et al., 2010), the process zone is represented by a discontinuity line of displacement inclined at an angle α to the interface. The stress state in the zone is defined in the polar coordinate system with the origin at the tip of the crack by the components of tensile and shear stresses on the discontinuity lines, σ_θ, $\tau_{r\theta}$ that satisfy the strength criterion of Mises-Hill type
$$\left(\frac{\sigma_\theta}{\sigma_1}\right)^2 + \left(\frac{\tau_{r\theta}}{\tau_1}\right)^2 = 1 \tag{1}$$
σ_1, τ_1 are the tensile and shear strength of the first material which are determined experimentally as the average values of normal and tangential stresses in the zone under the corresponding mode of loading of a homogeneous material.
3. Limiting ourselves to the study of the stress state in the vicinity of the crack tip at the stage of the process zone initiation, when its length is significantly shorter than the length of the crack, we will consider the body as a piecewise homogeneous plane with a semi-infinite cut at the interface and a displacement discontinuity line of finite length that begins at the tip of the cut and extends into the first material at an angle α to the interface (Fig. 1). We assume that the process zone size is such that there exists a range $l \ll r \ll L$. In this range, it is possible to neglect the process zone influence on the stressed state, i.e. it is possible to use the solution for the problem without process zone. This assumption allows us to use Prandtl's method of asymptotic splicing of asymptotic expansions (Nayfeh, 2000) to formulate a condition at infinity in the form of a requirement for the transition of the desired solution of the problem at the distances to the asymptotics of the solution of a similar problem of the elasticity theory without the discontinuity line. We present this asymptotics within the framework of the open crack model (Williams, 1959; England, 1965; Erdogan, 1965; Rice and Sih, 1965) in the normalized form proposed by Rice (1988).

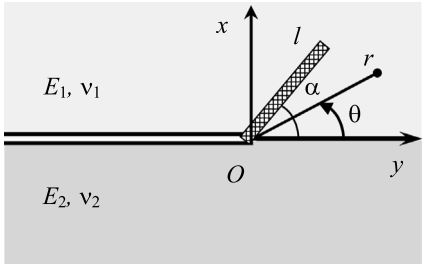


Fig. 1. Piecewise-homogeneous body model with interface crack and process zone.

As a result, as the faces of the crack are free of loading, taking into account the condition (1), we arrive at the static boundary value problem of the theory of elasticity with boundary conditions

$$\theta = -\pi \text{ and } \theta = \pi : \sigma_\theta = \tau_{r\theta} = 0; \tag{2}$$

$$\theta = 0 : \langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0, \langle u_\theta \rangle = \langle u_r \rangle = 0; \tag{3}$$

$$\theta = \alpha, r < l : \sigma_\theta = \sigma_1 \cos\psi(r), \tau_{r\theta} = \tau_1 \sin\psi(r),$$

$$\theta = \alpha, r > l : \langle u_\theta \rangle = \langle u_r \rangle = 0; \tag{4}$$

$$\theta = \alpha, r \to \infty : \sigma_\theta \sim 2\mathrm{Re}\left[CF_1(\alpha) r^{\lambda_0}\right] + o(1/r), \tau_{r\theta} \sim 2\mathrm{Re}\left[CF_2(\alpha) r^{\lambda_0}\right] + o(1/r). \tag{5}$$

Here $\langle f \rangle$ denotes a jump in the value of f; $\lambda_0 = -0.5 + i\omega$ is the complex power of stress singularity near the tip of an open interface crack with an imaginary part that causes spatial oscillations of stresses and displacements;

$$\omega = \frac{1}{2\pi} \ln \frac{1-\beta}{1+\beta}, \quad \beta = \frac{(1+e\kappa_2) - (e+\kappa_1)}{(1+e\kappa_2) + (e+\kappa_1)}, e = \frac{1+\nu_2}{1+\nu_1}\frac{E_1}{E_2}, \kappa_{1(2)} = 3 - 4\nu_{1(2)};$$

$$C = -ie' K L^{-i\omega}, e' = \frac{\sqrt{(e+\kappa_1)(1+e\kappa_2)}}{2\sqrt{2\pi}[(e+\kappa_1)^2 - (1+e\kappa_2)^2]},$$

$$F_1(\alpha) = -\left[a_1 \sin(\lambda_0+2)\alpha + a_2 \sin\lambda_0\alpha + a_3 \cos(\lambda_0+2)\alpha + a_4 \cos\lambda_0\alpha\right],$$

$$F_2(\alpha) = a_1 \cos(\lambda_0+2)\alpha + a_2\lambda_0 \cos\lambda_0\alpha - a_3 \sin(\lambda_0+2)\alpha - a_4\lambda_0 \sin\lambda_0\alpha,$$

$$a_1 = \frac{e+\kappa_1}{2} - 1 - e\kappa_2 + \left(e + \kappa_1 - \frac{1+e\kappa_2}{2}\right)\cosh 2\pi\omega + i\omega\left[e + \kappa_1 - (1+e\kappa_2)\cosh 2\pi\omega\right],$$

$$a_2 = \left(\frac{3}{2} + i\omega\right)\left[(1+e\kappa_2)\cosh 2\pi\omega - e - \kappa_1\right],$$

$$a_3 = -\omega(1+e\kappa_2)\sinh 2\pi\omega - i\left(e + \kappa_1 - \frac{1+e\kappa_2}{2}\right)\sinh 2\pi\omega,$$

$$a_4 = \left(\omega - \frac{3i}{2}\right)(1+e\kappa_2)\sinh 2\pi\omega;$$

β is the second Dundurs parameter (Dundurs, 1969), $|\beta| \le 0.5$; $K = K_1 + iK_2$ is complex stress intensity factor, which characterizes the magnitude and configuration of the external load. In what follows, we consider K as specified by the problem statement and as such, ensures the tensile loads in the process zone.

The parameter $\psi(r)$, introduced in (4) to linearize condition (1), determines the ratio of tangential and normal stresses in the zone $(\tau_{r\theta}(r,0)/\sigma_\theta(r,0) = \tau_1/\sigma_1 \cdot \tan\psi(r))$ and is close in meaning to the phase angle of stresses, for which different definitions take place — due to the ratio of stresses, intensity coefficients, etc. Rice (1988). In what follows, taking into account that the process zone is small, we neglect the dependence of ψ on r and consider it equal to the average value of the phase angle of stresses in the zone: $\psi(r) \equiv \psi = \text{const}$. The determination of the angle ψ, as well as the length of the process zone l, is carried out when there is no singularity of stresses at the tip of the zone, while the angle of zone inclination α will be determined from the condition of the maximum amount of energy dissipation in the zone.

Near the end of the discontinuity line, in accordance with the general provisions on the behavior of stresses and displacements in elastic bodies in the vicinity of sharp concentrators (Cherepanov, 1979; Parton and Perlin, 1984), asymptotics follows the largest solution of a homogeneous problem of the elasticity theory for a plane containing a straight displacement line. In particular, the following asymptotics apply

$$\sigma_\theta(r,\alpha) \sim \frac{k_1}{\sqrt{2\pi(r-l)}} \ (r \to l+0), \left\langle \frac{\partial u_\theta}{\partial r} \right\rangle \sim -\frac{4(1-\nu_1^2)}{E_1}\frac{k_1}{\sqrt{2\pi(l-r)}} \ (r \to l-0),$$

$$\tau_{r\theta}(r,\alpha) \sim \frac{k_2}{\sqrt{2\pi(r-l)}} \ (r \to l+0), \left\langle \frac{\partial u_r}{\partial r} \right\rangle \sim -\frac{4(1-\nu_1^2)}{E_1}\frac{k_2}{\sqrt{2\pi(l-r)}} \ (r \to l-0), \tag{6}$$

where the stress intensity factors k_1, k_2 in the end of the displacement line are to be determined.

3. Problem solution technique

3.1. Reduction to the vector Wiener–Hopf equation

In accordance with the principle of superposition, the solution of the formulated boundary value problem of the elasticity theory is given in the form of the sum of the following two problems solution. The first differs from it in that in (4) instead of the first pair of conditions is replaced with

$$\theta = \alpha, r < l : \quad \sigma_\theta = \sigma_1 \cos\psi(r) - 2Re\left[CF_1(\alpha) r^{\lambda_0}\right],$$

$$\tau_{r\theta} = \tau_1 \sin\psi(r) - 2Re\left[CF_2(\alpha) r^{\lambda_0}\right], \tag{7}$$

at the same time, at infinity, the stresses and relative strains decrease as $o(1/r)$. The second problem is a similar problem without the discontinuity line, the solution of which is known (Rice, 1988), so it is sufficient to find the solution of the first problem.

By applying Mellin's integral transformation of the form $m^*(p) = \int_0^\infty m(r) r^p dr$ to the equilibrium equations, the condition of compatibility of deformations, the Hooke's law and the boundary conditions (2)–(3) and using the second part of the conditions (4) and the condition (7), we arrive at the system of functional equations (the Wiener–Hopf vector equation) of the first problem for the transformants of stresses and displacement gradients in the strip $-\varepsilon_1 < \mathrm{Re}\ p < \varepsilon_2$ (ε_1, ε_2 are sufficiently small positive numbers):

$$\boldsymbol{\Phi}_1^+(p) + \frac{\sigma_1 \cos\psi}{p+1} + \frac{Z_1}{p+0.5+i\omega} + \frac{\bar{Z}_1}{p+0.5-i\omega} = G_{11}(p)\boldsymbol{\Phi}_1^-(p) + G_{12}(p)\boldsymbol{\Phi}_2^-(p),$$

$$\boldsymbol{\Phi}_2^+(p) + \frac{\tau_1 \sin\psi}{p+1} + \frac{Z_2}{p+0.5+i\omega} + \frac{\bar{Z}_2}{p+0.5-i\omega} = G_{21}(p)\boldsymbol{\Phi}_1^-(p) + G_{22}(p)\boldsymbol{\Phi}_2^-(p), \tag{8}$$

$$\boldsymbol{\Phi}_1^+(p) = \int_1^\infty \sigma_\theta(\rho l, \alpha)\rho^p d\rho, \quad \boldsymbol{\Phi}_1^-(p) = \frac{E_1}{4(1-\nu_1^2)l}\int_0^1 \left\langle \frac{\partial u_\theta(\rho l, \alpha)}{\partial \rho} \right\rangle \rho^p d\rho,$$

$$\boldsymbol{\Phi}_2^+(p) = \int_1^\infty \tau_{r\theta}(\rho l, \alpha)\rho^p d\rho, \quad \boldsymbol{\Phi}_2^-(p) = \frac{E_1}{4(1-\nu_1^2)l}\int_0^1 \left\langle \frac{\partial u_r(\rho l, \alpha)}{\partial \rho} \right\rangle \rho^p d\rho,$$

$$Z_1 = -CF_1(\alpha) l^{\lambda_0}, Z_2 = -CF_2(\alpha) l^{\lambda_0};$$

$$G_{mn}(p) = -\tan p\pi G_{mn}^0(p) \ (m,n = 1,2), G_{mn}^0(p)$$