DOI 10.1007/s10778-022-01165-y
International Applied Mechanics, Vol. 58, No. 4, July, 2022

ON THE SCOPE AND LIMITATIONS OF THE COMNINOU MODEL FOR A CRACK AT A POLYGONAL INTERFACE

A. O. Kaminsky[1*], M. V. Dudyk[2], V. M. Fenkiv[2], Y. O. Chornoivan[1]**

A plane strain problem is investigated for the local stress and displacement fields near the corner point of the polygonal interface of two dissimilar homogeneous isotropic materials. It is supposed that the interface is weakened by an interface crack with the tip at the corner point. The contact zone near the crack tip is modeled by a cut with frictional contact between the faces according to the Comninou model. The asymptotic expressions for the components of the displacement gradients and stress tensor near the crack tip are found using the Wieghardt–Williams method. A characteristic equation is obtained to determine the singularity order at the crack tip. The dependence of the singularity order on the angle of the interface and the ratio of Young's moduli of the bounding materials is analyzed numerically. It is shown that the Comninou model can be used in this problem to eliminate the displacement oscillations near the corner point of the interface between two dissimilar materials. The obtained results are used to determine the scope of model applicability.

Keywords: fracture, interface crack, polygonal interface, Wieghardt–Williams method, singularity order, contacting faces

1. Introduction. Solving many problems of fracture mechanics requires knowledge of the stress–strain state near various natural defects in the structure of solids and near sharp technological elements of structures, which are a source of high-stress levels. When calculated within the framework of the linear elasticity theory, local stresses are singular in the vicinity of the concentrator vertices. The foundations of linear fracture mechanics were laid in the last century by A. Griffith, G. R. Irwin, and E. Orowan. The fracture criteria formulated by them ensured significant progress in the mechanics of fracture, first of all, of homogeneous bodies with cracks.

The next step in the development of fracture mechanics was to take into account the influence of pre-fracture zones on the stress–strain state near the crack tips. To this end, more complex models of cracked bodies have been developed, in particular, the models of Barenblatt [6] and Dugdale [17] and their generalization [9, 26 – 30], which describe the prefracture zone of by rupture lines.

At the same time, studies of piecewise-homogeneous bodies with a crack at the flat interface between two dissimilar materials carried out by Williams [42], Erdogan [20], England [19], Rice and Sih [37], and others resulted in physically impossible mutual intersection of crack edges due to spatial oscillations of the displacement in bounded bodies near the vertices. Further investigations showed that oscillatory singularities at the tip of an open interface crack cannot be eliminated by taking into account additional factors, namely the curvilinear form of the interface crack [43], spatial configuration of the piecewise homogeneous body [7] and the anisotropy of the materials [36]. The oscillatory singularity of the displacement field in the vicinity of the crack tip is an important issue in thermoelasticity [31] and piezoelectricity in bimaterials with impermeable and permeable interface crack [21].

[1]S. P. Timoshenko Institute of Mechanics, National Academy of Sciences, 3 Nesterov St., Kyiv, Ukraine, 03057; *e-mail: fract@inmech.kiev.ua. [2]Pavlo Tychyna Uman State Pedagogical University, 2 Sadova St., Uman, Cherkassy Region, Ukraine, 20300; **e-mail: dudik_m@hotmail.com. Published in Prykladna Mekhanika, Vol. 58, No. 4, pp. 32–44, July–August 2022. Original article submitted July 10, 2021.

 1063-7095/22/5704-0398 ©2022 Springer Science+Business Media, LLC

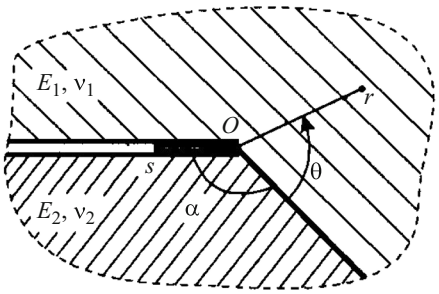


Fig. 1

Oscillating singularities at the tip of an open interface crack caused some difficulties with the correct application of the linear fracture mechanics criteria [23]. A solution to this problem was proposed by M. Comninou [11]. It supposes a face contact area near the tip of an interface crack. Investigations by Comninou [10, 11], Comninou and Dundurs [12], Dundurs and Gautesen [18], Anderson [1], Antipov et al. [3], Atkinson [4], Audoly [5], Leguillon [32] and others did confirm the elimination of displacement oscillations and the preservation of the power-law stress singularity due to the contacting faces for the planar interface between isotropic materials. The contact model was successfully used to investigate planar interfaces of various anisotropic, piezoelectric, electrically permeable, and dielectric materials in various configurations [21, 22, 24, 25, 39].

But is the Comninou model perfect? Does it guarantee that there will be no oscillatory singularity for every interface crack? To answer this question, we will consider a crack at a piecewise-linear interface. As was stated earlier, the vast majority of interface crack studies deal with the case where the crack is located on the planar interface between two dissimilar media. However, in the structure of modern industrial products or rocks, cracks may appear at a broken interface. Insufficient attention has been paid to the determination of the stress–strain state in the vicinity of the tips of such cracks. In particular, Theocaris and Gdoutos [38], using the Muschelishvili complex potential method, obtained explicitly characteristic equations for stress singularity orders within the first, second, and mixed basic problems of the theory of elasticity for an interface crack emerging from the corner point of two dissimilar homogeneous materials neglecting possible the contact between the crack faces.

Under certain assumptions, the stress singularity orders under the above conditions can be determined from the results of local stress field studies in the vicinity of the common vertex of a multiwedge heterogeneous system [8, 13, 33–35]. In work [15], an analysis of the stress field near the tip of an open crack emerging from the corner point of the interface was performed. Following the results of Theocaris and Gdoutos [38], it is shown that in a rather wide range of fracture angles the stress singularity order is complex. As in the case of an interface crack at the planar interface, this yields spatial oscillations of the crack face displacement. As mentioned above, an alternative to the model of an open interface crack is the model of a crack with contacting faces (Comninou model), in which, as in the model of M. Comninou, the displacement oscillation can be eliminated. In this regard, it is important to study the local field of stresses and displacements near the corner point of the boundary between two dissimilar materials from which the interface crack emerges. The faces are supposed to interact according to the law of dry friction. The problem is to find the scope of applicability of the model and the asymptotic expressions for the components of the stress tensor near the crack tip under the conditions of plane strain. These asymptotic expressions can be used to determine the limiting states of piecewise-homogeneous bodies with an interface crack, the cohesive zone parameters at the crack tip, etc.

The general notation and the problem statement is given in Sect. 2. Section 3 is devoted to the solution of the problem. The solution and its implications are discussed in Sect. 4. The corresponding asymptotical near-tip stress field is analyzed in Sect. 5 with some necessary functions given in Appendix A.

2. Problem Statement. Consider the behavior of the stresses near the tip of an interface crack under the plane strain condition. The tip of the crack is located at the corner point of the polygonal boundary between two dissimilar elastic homogeneous isotropic materials. The materials are characterized by Young's moduli E_1, E_2 and Poisson's ratios ν_1, ν_2, respectively. It is assumed that part of the crack faces of length s is in Coulomb frictional contact (Fig. 1). The coefficient of friction is μ. Following the general stress behavior near the corner points of elastic bodies (at distances $r << s$), we arrive at a homogeneous problem of elasticity theory for a piecewise-homogeneous plane with a boundary between materials in the form of rectilinear sides of an angle. A half-infinite crack with contacting faces emerges from the angle vertex. The local structure of a piecewise-homogeneous body with an interface crack is depicted in Fig. 1.

In what follows, the polar coordinate system (r,θ) is introduced in such way that the pole O is at the vertex of the angle of measure α at the interface and the polar axis is located along the interface line of materials. Taking into account the contact of the crack edges and the condition of continuity of stresses and displacements at the interface line of materials, one can formulate the boundary-value problem of the theory of elasticity as follows:

$$\sigma_\theta^1(r,0)=\sigma_\theta^2(r,0),\qquad \tau_{r\theta}^1(r,0)=\tau_{r\theta}^2(r,0),$$

$$u_\theta^1(r,0)=u_\theta^2(r,0),\qquad u_r^1(r,0)=u_r^2(r,0),$$

$$\sigma_\theta^1(r,\beta)=\sigma_\theta^2(r,-\alpha),$$

$$\tau_{r\theta}^1(r,\beta)=\tau_{r\theta}^2(r,-\alpha)=-\mu\sigma_\theta^1(r,\beta),$$

$$u_\theta^1(r,\beta)=u_\theta^2(r,-\alpha)\qquad(\beta=2\pi-\alpha), \tag{1}$$

where the superscripts denote the number of material, σ_r, σ_θ, $\tau_{r\theta}$ are the radial and tangential components of the stress tensor, and u_r, u_θ are the components of the displacement vector.

3. Determination of Stress Singularity Orders. Since the studied body is a combination of two elastic wedges, we use the Wieghardt–Williams method [40, 41] to expand the solutions of the elasticity equations into series of eigenfunctions and represent the stresses and derivatives of displacements in each of the materials as

$$\sigma_\theta^n(r,\theta)=r^\lambda[a_1^n\sin(\lambda+2)\theta+a_2^n\sin\lambda\theta+a_3^n\cos(\lambda+2)\theta+a_4^n\cos\lambda\theta],$$

$$\tau_{r\theta}^n(r,\theta)=-\frac{r^\lambda}{(\lambda+2)}[a_1^n(\lambda+2)\cos(\lambda+2)\theta+a_2^n\lambda\cos\lambda\theta-a_3^n(\lambda+2)\sin(\lambda+2)\theta-a_4^n\lambda\sin\lambda\theta],$$

$$\sigma_r^n(r,\theta)=-\frac{r^\lambda}{\lambda+2}[a_1^n(\lambda+2)\sin(\lambda+2)\theta$$

$$+a_2^n(\lambda-2)\sin\lambda\theta+a_3^n(\lambda+2)\cos(\lambda+2)\theta+a_4^n(\lambda-2)\cos\lambda\theta],$$

$$\frac{\partial^2u_\theta^n(r,\theta)}{\partial r^2}=-\frac{1+\nu_n}{E_n}\frac{\lambda r^{\lambda-1}}{\lambda+2}[a_1^n(\lambda+2)\times\cos(\lambda+2)\theta+a_2^n(\lambda+4-4\nu_n)\cos\lambda\theta$$

$$-a_3^n(\lambda+2)\sin(\lambda+2)\theta-a_4^n(\lambda+4-4\nu_n)\sin\lambda\theta],$$

$$\frac{\partial u_r^n(r,\theta)}{\partial r}=-\frac{1+\nu_n}{E_n}\frac{r^\lambda}{\lambda+2}[a_1^n(\lambda+2)\sin(\lambda+2)\theta$$

$$+a_2^n(\lambda-2+4\nu_n)\sin\lambda\theta+a_3^n(\lambda+2)\cos(\lambda+2)\theta+a_4^n(\lambda-2+4\nu_n)\cos\lambda\theta], \tag{2}$$

$n=1$ for $0\le\theta\le\beta$ and $n=2$ for $-\alpha\le\theta\le0$. Satisfying the boundary conditions (1) using Eqs. (2), we arrive at a system of linear homogeneous algebraic equations for the coefficients a_j^n $(j=1\ldots4)$. For this system to have a nontrivial solution, its determinant must be zero. This condition leads to the characteristic equation of the problem for determining the exponent λ in the functional dependence of stresses on the distance r to the crack tip. After calculating the determinant and its algebraic transformations, this equation gives

$$D(\lambda)=0,\qquad D(\lambda)=D_1(\lambda)+\mu D_2(\lambda), \tag{3}$$

where

$$D_1(\lambda)=(1-e)[0.5(1+\kappa_1)D_{11}t_1-0.5e(1+\kappa_2)D_{12}t_3+e(1+\kappa_1)(1+\kappa_2)d_3]+e(1+\kappa_1)(1+\kappa_2)D_{13},$$