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ANALYTICAL METHODS OF MODELING THE PREFRACTURE ZONE NEAR AN INTERFACIAL CRACK TIP (REVIEW)

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A review of analytical models of the prefracture zone in the vicinity of the tip of an interfacial crack in a piecewise-homogeneous elastic-brittle body is given in connection with the study of crack rotation from the interface of the connected materials. They are based on a complex model of an interfacial crack, which assumes the presence in the vicinity of the crack tip of a contact area between the faces and a small-scale prefracture zone with a near-the-tip domain of material destruction. The main provisions of the analytical method for taking into account the small-scale prefracture zone near the tip of an interfacial crack are stated, they allow us to reduce the initial nonlinear problem of fracture mechanics to solving the boundary value problem of the elasticity theory about the displacement discontinuity line extending from the crack tip into a less crack-resistant material. The mathematical apparatus for analytically solving the boundary value problems of the analyzed models is the Wiener–Hopf method in combination with the Mellin integral transform. Variants of implementation of the complex model of an interfacial crack for the cases of predominant tension or shear are considered. A numerical analysis of the parameters of the prefracture zone and the area of face contact for different loading conditions was performed. The predictions of the model are compared with the experimental results of the study of interfacial crack rotation. The mechanism of opening of an initially closed crack due to the formation of the destruction zone is described. The results of an approximate analytical solution to the problem of a complex model of an interfacial crack under loads that provide commensurate zones of face contact and prefracture are analyzed. The mutual effect of the contact of the faces and the prefracture zone on their parameters and the angle of rotation of the crack was determined.

Keywords: interface crack, complex model, process zone, destruction zone, contact of the faces, crack kinking

Introduction. The problem of interfacial crack rotation has been actively studied over the past few decades, but it is still of interest nowadays. The relevance of this problem remains high due to the widespread use of composites, laminates, and other types of piecewise-homogeneous materials in modern manufacturing, which leads to problems on the origination and further evolution of interfacial cracks in them.

The study of interfacial cracks began in the second half of the last century with the determination of the local stress and displacement field in the vicinity of its tips. In the theory of interfacial cracks, the most popular are the model of an open crack-cut with stress-free faces [28, 29, 54, 60] and the Comninou model [17–20, 27], which postulates the presence of contact of the faces near the crack tip. However, none of these models is fully satisfactory. The open crack model leads to an oscillatory singularity of stresses and strains when approaching the tip, which results in a physically impossible mutual intersection of the faces. On the other hand, the contact model, under certain loading conditions (e.g., when tension prevails), predicts extremely

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small dimensions of the contact domain of the faces, which casts doubt on the existence of the contact. Accordingly, the choice of a basic model of the interfacial crack for predicting its propagation direction should be primarily determined by the loading conditions of the piecewise homogeneous body. This, accordingly, will affect the statement of the crack rotation problem.

The standard method of calculating the crack rotation angle is reduced to solving the problem of a small-scale crack propagating from the tip of the initial interfacial crack into one of the materials of the composite joint [34]. Different criteria for choosing the orientation of the rotated crack can be applied to the obtained solution, among which the most commonly used, effective, and close in terms of the predicted values of the rotation angles are the condition of the maximum energy release rate and the principle of local symmetry (local stress intensity factor near the tip of the secondary lateral crack $k_{11} = 0$ [62].

The method is based on the study of only the last stage of crack rotation when the rotation has already happened. It does not take into account the processes of material fracture that precede the formation of the rotated crack and does not explain the mechanism of opening of an initially closed interfacial crack within the model with contacting faces. In this regard, it is expected that consideration of the initial stage of crack rotation, which consists of the formation of stress concentrators and prefracture zones in the material into which the crack rotates, will allow a better understanding of the features of the crack rotation process under different loading conditions and clarify the conditions of displacement and the expected values of the angles of crack deviation from the interface.

Due to the mathematical complexity, the number of works devoted to the near-the-tip prefracture zones near interfacial cracks and their effect on crack rotation is rather small, and the calculations of the parameters of the zones were performed mainly by numerical methods [13, 52]. An exception is the series of studies [2, 3, 22–24, 36, 38, 39, 41] of small-scale prefracture zones performed by the authors of this publication with the active participation of one of the inspirers of these studies, Professor L. A. Kipnis and his other colleagues. The model representation of prefracture zones in them by a mathematical cut (displacement discontinuity line) with a certain law of interaction of its faces allowed the authors to reduce the initial problem of interfacial crack rotation under different loading conditions to the corresponding boundary value problems of elasticity theory solved analytically using the Wiener–Hopf method [49].

The purpose of this publication is to review the analytical models of near-tip prefracture zones, developed by the authors and the prospects for their further improvement and to substantiate the possibilities of their use in connection with interfacial crack rotation. The diversity and number of analyzed models are due, on the one hand, to the significant effect of the nature of the external load on the possible ratios of the sizes of prefracture zones to the contact domain between the faces, which leads to the multiscale of the structural members of the models and creates difficulties for the development of one universal model, and, on the other hand, is associated with an attempt to take into account the presence of a rather small domain of material destruction in the part of the prefracture zone adjacent to the crack tip, which is described by an increased level of deformation.

1. Main Provisions of the Method of Accounting for Small-Scale Prefracture Zones in the Theory of Interfacial Cracks. The method of taking into account the effect of small-scale prefracture zones on the stress-strain state in the vicinity of sharp-edged stress concentrators in plane problems of fracture mechanics, which is used in the following sections of the article, was first implemented by Cherepanov in [16] when calculating the parameters of the plastic zone near the tip of a normal tear crack in a homogeneous elastic-plastic material. The method is based on the following provisions:

1. Decomposition of the initial problem of determining the parameters of a small-scale prefracture zone (PZ) and stress–strain state (SSS) in the vicinity of the tip of a sharp-edged stress concentrator into an external problem describing the SSS of the body without PZ and an internal problem aimed at determining the parameters of PZ and SSS near the tip.

The decomposition of the original problem into external and internal problems in the case of small-scale PZ makes it possible to solve them independently. This is important because the solution of the external problem is significantly complicated in the case of a non-trivial structure of the body and a complex configuration of the external load, while the internal problem can be solved separately within the framework of linear fracture mechanics only if the asymptotic solution of the external problem is known.

2. Use of the "microscope principle" [11] to describe the local stress and displacement field near the tip of the concentrator.

To solve the internal problem in the case of a small-scale PZ, it is enough to know the local stress field near the tip of the stress concentrator as an asymptotic part of the solution to the external problem. The "microscope principle" allows us to reduce the search for the asymptotic part of the solution of the external problem to solving the corresponding homogeneous canonical singular boundary value problem of elasticity for special points, which are sharp-edged concentrators, in particular, crack tips.

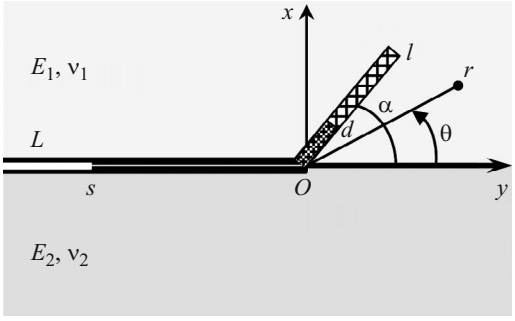


Fig. 1

The solution to this problem is found using the Wieghardt–Williams method of eigenfunction expansion [58, 59]. Within the framework of the Wieghardt–Williams method, the asymptotic solution is represented as a superposition of partial solutions

$$\sigma_{jk}(r,\theta) \sim \sum_i C_i F_{jk}(\lambda_i,\theta) r^{\lambda_i} \qquad (r \to 0) \tag{1.1}$$

(r, θ are the coordinates in the polar system with the origin at the tip of the stress concentrator), the contribution of each of which is determined by:

– coefficients C_i that depend on the structure of the body and external load and are calculated by numerical or, if possible, analytical methods from the solution of the external problem. Physically, C_i correspond to stress intensity factors. It is important that in solving the internal problem, there is no need to determine the coefficients C_i: at this stage, it is sufficient to consider them as given, and their numerical values for a specific body and a specific load are only required for numerical analysis of the obtained solution and substituted into the final formulas;

– the roots λ_i ($\mathrm{Re}\,\lambda_i > -1$) of the characteristic equation of the problem, which are associated with the local structure of the near-the-tip domain and, in the case of negative values of their real parts, are singularity exponents of stresses. The constraint $\mathrm{Re}\,\lambda_i > -1$ is due to the requirement of finite displacements when $r > 0$;

– functions $F_{jk}(\lambda_i, \theta)$, which, like singularity exponents λ_i, are determined by the local geometry and elastic characteristics of materials in the near-the-tip domain.

3. Using the Leonov–Panasyuk [6, 7] and Dugdale [26] models to reduce the internal problem of calculating the parameters of the PZ to a static boundary value problem of the theory of elasticity about the displacement discontinuity line as a mathematical cut extending from the tip of the concentrator with a given law of interaction of its faces that satisfies a certain criterion of material strength.

Determination of the shape and size of the prefracture zone within the framework of nonlinear fracture mechanics involves solving a system of partial differential equations with boundary conditions on a previously unknown boundary that separates the PZ from the rest of the material in an elastic state and satisfies the equations of the theory of elasticity. Nowadays, due to the lack of effective methods for analytical solutions of such nonlinear problems, their solutions are mostly numerical. However, in the case of a small-scale prefracture zone, a high strain gradient and a rapid increase in stresses as one approaches the tip of the concentrator make it difficult to apply numerical methods. The Leonov–Panasyuk and Dugdale models avoid this problem by removing the prefracture zone from the body, instead connecting the boundaries of the removed domain under the assumption of the existence of a displacement jump at the boundary interface line and stating certain stress conditions on it that appropriately represent the state of the material in the prefracture zone.

The problem of the initial stage of interfacial crack rotation under plane deformation in the general statement that takes into account the contact of the faces, the formation of a prefracture zone with a domain of material destruction in its near-the-tip part was stated in [3] within the framework of the so-called complex model. The representation of the prefracture zone by a displacement discontinuity line leads to the design model shown in Fig. 1. The rectilinear section OO' models a prefracture zone of length d containing a destruction zone OO'' of length d. The limitation to studying the SSS only in the vicinity of the tip of the interfacial crack and the assumption of a small-scale prefracture zone (the length of the prefracture zone l is much smaller than the length of the crack L allows us to consider the body as a piecewise homogeneous plane with a semi-infinite cut at the interface