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INVESTIGATION OF THE INITIAL STAGE OF FRACTURE OF A PIECEWISE HOMOGENEOUS BODY WITH INTERFACE CRACK COMPRESSED ALONG THE INTERFACE

A. O. Kamins'kyi,[1] **M. V. Dudyk,**[2,3] **Yu. V. Reshitnyk,**[2] **and V. M. Fen'kiv**[2]

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We consider the initial stage of the fracture process in a piecewise homogeneous body compressed along the plane interface of materials containing an interface crack. We perform analytic evaluation of the parameters of a small-scale prefracture zone in the joining material near the tips. The values of separation of the boundaries of this zone are determined. This quantity is used in the deformation criterion of formation of a secondary microcrack from this zone. The mechanism of crack-growth initiation is proposed for an incipient crack. This mechanism leads to merging of this crack with a growing microcrack under increasing loading.

Keywords: interface crack, compression along the interface, contact of faces, prefracture zone, secondary microcrack, mechanism of crack-growth initiation.

Introduction

As one of urgent problems of the mechanics of deformable solid, we can mention the problem of investigation of the mechanism of fracture processes in bodies subjected to compression. The problem of crack-growth initiation under the conditions of compression of a body in the direction parallel to the plane of the crack belongs to this class of problems. Under the indicated loading, singular components are absent in the field of stresses near the crack tips (stress intensity factors are equal to zero), which excludes the possibility of application of the criteria of linear fracture mechanics.

The results of investigations of the behavior of cracks under the conditions of compression of the body parallel to the plane of their location show that, in this case, fracture of the body is caused by the local loss of the equilibrium state near the crack tips [2, 12]. This loss of stability may lead to the deflection of crack faces in different directions, in one direction with contact of the faces, or in one direction with formation of a gap between the faces. These processes lead to stress concentration and the formation of prefracture zones in the vicinity of crack tips, which enables us to apply, in this stage, traditional methods of investigation of the conditions of crack-growth initiation within the framework of linear fracture mechanics.

In the present paper, we consider the initial stage of the fracture process in a piecewise homogeneous body subjected to compression along the plane interface of materials, which contains an interface crack. Under the assumption that the loss of stability of the equilibrium state leads to the combined bending of the crack with contact of its faces to the side of less stiff material, we compute the parameters of small-scale prefracture zone

[1] S. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, Kyiv, Ukraine.

[2] P. Tychyna Uman State Pedagogic University, Uman, Ukraine.

[3] Corresponding author; e-mail: dudik_m@hotmail.com.

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formed in the joining material in the vicinity of the crack tips. We determine the value of separation of the boundaries of prefracture zone and then use it in the deformation criterion of initiation of a secondary microcrack in this zone. We propose a mechanism of crack-growth initiation for an incipient crack, which guarantees merging of this crack with a growing microcrack as the load increases. A similar mechanism of interface crack-growth initiation under the action of shear loads was described in [6].

1. Statement of the Problem

Under the conditions of plane deformation, we consider a problem of determination of the stress-strain state in the vicinity of the tip of an interface crack located on the plane interface of two different homogeneous isotropic elastic materials with Young's moduli E_1 and E_2 and constant Poisson's ratios ν_1 and ν_2, respectively. The investigated piecewise homogeneous body is subjected to compression in the direction parallel to the plane of joint and the crack faces. This leads to bending of the crack faces to the side of less stiff material (for definiteness, this is the first material, $E_1 < E_2$), the appearance of stress concentration, and the formation of prefracture zones in a thin layer of joining material in the vicinity of the crack tips; this material is less crack-resistant than the materials of joined parts of the body (Fig. 1a). Assume that the applied loads do not violate linearity of the interface outside the crack. In the case of uniformly distributed compressive loading applied to the end faces of the body, it is necessary to require the validity of the following condition established by Rice and Sih in [14]:

$$p_2 = \frac{E_2}{E_1}\frac{1-\nu_1^2}{1-\nu_2^2}\,p_1.$$

In the initial stage of loading, the sizes of prefracture zones are small. Therefore, we can assume that the length ℓ of each zone is much smaller than the crack length L. This enables us to restrict ourselves to the investigation of the stress-strain state solely of a local domain in the vicinity of one of the tips and reduce the original problem to the problem of determination of the parameters of a small-scale prefracture zone in the joining material near the tip of a semiinfinite interface crack in the piecewise uniform plane originating from the angular point of the broken interface (Fig. 1b).

Since the crack propagates according to the brittle mechanism, we simulate the prefracture zone by a straight line of discontinuity of normal displacements, where the normal stresses are equal to the tensile resistance of the joining material σ_0. Assuming that the contact of crack faces is realized with friction according to the Coulomb law, we arrive at the static boundary-value problem of the theory of elasticity with the following boundary conditions:

$$\theta = 0: \quad \langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0, \quad \langle u_r \rangle = 0,$$

$$\theta = -\alpha \cup (2\pi - \alpha): \quad \langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0, \quad \langle u_\theta \rangle = 0, \quad \tau_{r\theta} = -\mu\sigma_\theta, \tag{1}$$

$$\theta = 0, \quad r < \ell: \quad \sigma_\theta = \sigma_1,$$

$$\theta = 0, \quad r > \ell: \quad \langle u_\theta \rangle = 0, \tag{2}$$

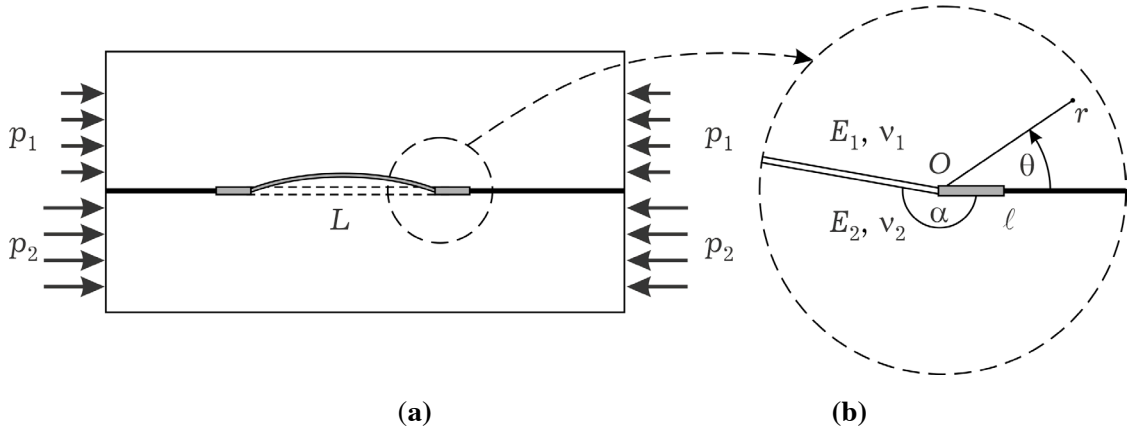


Fig. 1. (a) Piecewise homogeneous body with interface crack under the conditions of compression; (b) computational schematic diagram of the problem.

where $\langle f \rangle$ is the jump of the quantity f and μ is the coefficient of friction between the crack faces.

To formulate conditions at infinity, we apply the method of joining of asymptotic expansions [13] according to which the required solution is joined at distances $\ell \ll r \ll L$ with a known solution of the similar problem of the theory of elasticity without prefracture zone [3]:

$$\theta = 0, \quad r \to \infty: \quad \sigma_\theta = CF(\alpha) r^\lambda + o\left(\frac{1}{r}\right), \tag{3}$$

where

$$F(\alpha) = (1-e)^2 f_1 + (1-e)(1+\kappa_2) f_2 + e(1-e)(1+\kappa_2) f_3$$

$$+ e(1+\kappa_1)(1+\kappa_2) f_4 + e^2 (1+\kappa_2)^2 f_5,$$

$$f_1 = -4\mu\delta_{11} t_1, \qquad f_2 = 4 t_1 [\cos\lambda\beta - \mu \sin(\lambda+2)\beta],$$

$$f_3 = \delta_{11}\delta_{32} - 2\lambda\delta_{22}\delta_1(\beta) + \mu[(\lambda+2)\delta_{12}\delta_{31} + 2\delta_{21}\delta_1(\beta) + 4\delta_{11}\delta_1(\alpha)],$$

$$f_4 = 2\sin\lambda\pi \big[2\sin\lambda(\pi-\alpha)\delta_1(\alpha) - \delta_{31}\cos\lambda(\pi-\alpha)$$

$$+ \sin(\lambda+2)\beta[\delta_{23}\sin\lambda(\pi-\alpha) + \lambda\delta_{22}\cos\lambda(\pi-\alpha) - 2\mu\delta_2]\big],$$

$$f_5 = 4\sin\lambda\pi\delta_2 [\cos(\lambda+2)\alpha + \mu\sin(\lambda+2)\alpha],$$

$$t_1 = (\lambda+1)^2 \sin^2\alpha - \sin^2(\lambda+1)\alpha,$$

$$\delta_{11} = \lambda\sin(\lambda+2)\beta - (\lambda+2)\sin\lambda\beta, \qquad \delta_{12} = \cos\lambda\beta - \cos(\lambda+2)\beta,$$