DOI 10.1007/s10778-025-01319-8
International Applied Mechanics, Vol. 60, No. 6, November, 2024

CRACK INITIATION MODEL IN ELASTIC PIECEWISE HOMOGENEOUS BODY WITH BROKEN INTERFACE*

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The model of a crack initiation in a piecewise homogeneous body with a broken interface between elastic isotropic materials joined by plastic adhesive is developed. The initial stage of the crack initiation under plane strain and symmetric load about the bisector of the break angle is considered as the formation of a plastic zone in the joining material and a process zone in one of the materials of the composite joining near the corner point of the interface. The parameters of the process zone are calculated and the conditions of the crack initiation in the zone are defined by the deformation and energy criteria. The results of numerical investigation of the parameters of the process zone are analyzed.

Keywords: piecewise homogeneous body, broken interface, plastic zone, process zone, crack initiation

Introduction. The adhesive joints of dissimilar materials with a broken interface frequently appear in industrial products. The presence of similar configuration of solids can be motivated both by the structural features of the products and the attempt to improve the strength of material joining due to additional resistance to fracture through compression of non-smooth surfaces as it holds in the case either doubled scarf joining or overlapping one [2, 12, 22]. For this reason, the necessity arises to study the possible processes of fracture of piecewise homogeneous bodies with the broken interface of joined materials. However, there are considerably less works devoted to the above theme than for the composite joints with a plane interface. Most of these works are addressed to the analysis of the singularity of stresses and determination of stress intensity factors near the corner point of the interface. Particularly, singularity indexes for such joints are calculated in [4, 9, 24, 33]. In [26], analytical expressions for a stress field are derived and results of the FEM-calculations of the stress intensity factors near the corner vertex of the broken interface under the action of thermal and mechanical loads are presented. The singularity indexes for composite joints of two anisotropic materials are studied in [8]. Wang, considering in [34] various kinds of bi-material systems, has analyzed how the singularity index depends on the interface break angle for a completely coupled bi-material joint under plane strain. Chen and Ping, using the hybrid method of finite elements, have calculated in [6] the singularity indexes and stress intensity factors near the corner points either of elastic rectangular or diamond-shaped inclusions in an elastic plane under the action of finitely distanced tensile and shear stresses. The studies of the stress-strain state near the mutual vertex of multi-wedge systems [5, 25, 31, 32], for which the bi-material joint with the interfacial crack emerging from the corner point of the broken interface is a partial case, are similar to the above-mentioned ones.

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* Based on the materials of the report at the International Scientific Conference "Modern Problems of Mechanics–2023" dedicated to the 145th anniversary of S. P. Timoshenko, Kyiv, November 14–16, 2023.

Nazarenko and Kipnis have studied in [28, 29] under the plane strain the stress-strain state of piecewise homogeneous body with the interface of media in the form of sides of the angle composed of isotropic elastic parts with allowance for presence of the small-scale shear interphase cracks emerging from an interface corner point. Particularly, in [28], the authors have defined the singularity index for stresses at the corner point and studied its dependency on the break angle and ratio of the elastic moduli of joined materials. In [29], under the load symmetric about the bisector of the break angle, the singularity index and stress intensity factor near the crack ends have been determined. Also, equations to design a fracture load have been derived and unstable nature of cracks equilibrium has been studied. The similar investigation of the stress state of piecewise isotropic elastic body near the small-scale mode 1 crack at the corner point of the media interface has been carried out in [20] and the stress intensity factor at the crack tip has been defined.

It should be noted that the above-mentioned studies do not contain the possible fracture mechanisms of piecewise homogeneous bodies with the broken interface and do not propose methods to calculate limiting loads in such bodies. As distinct from them, the works [19, 23] develop the models of interfacial fracture based on the concept of fracture mechanics that assume the formation of process zones near the vertices of stress concentrators. Works [19, 23] are based on the Leonov-Panasyuk [1] and Dugdale [11] models which are effectively applied in fracture mechanics of piecewise homogeneous bodies [15, 18, 21]. Particularly, in [23], with the assumption on the symmetry of the applied external load about the bisector of the break angle of the interface of the composite joint of two dissimilar elastic materials, the parameters of a small-scale plastic zone in a joining layer (adhesive) are calculated. Here, the plastic zone has been modeled by two symmetrical lines of discontinuity of the tangential displacement at which the stress is equal to the yield point of the joining material. Having solved the problem, the length of the plastic zone and local field of stresses in the vicinity of the interface corner point have been defined. In [19], similar investigation of the initial process zone in the joining brittle material near the corner point of the interface of elastic materials has been carried out. Similar models of the process zones either of brittle or plastic type near the corner point of the interface have been developed in [10, 16, 17]. Here, it is supposed that the mode 1 crack emerges from this point along the bisector of the break angle. The variants of formation of these zones both in adhesive and in one of the materials of the composite joint have been considered.

Not all models developed in [10, 16, 17, 19, 23] are fit to predict the fracture process of piecewise homogeneous body with the broken interface because the small-scale plastic zones in the body are insufficient to study the pure plastic fracture. Along with this, such zones can be considered as the accompanying elements of the body structure at the brittle character of failure of joined materials since they strongly influence on the stress state near the break angle. For this reason, this paper considers a problem on construction of the model of brittle failure of joined materials using plastic adhesive to joint them. It is assumed that the small-scale plastic zones arise at the initial stage in the joining material. Their parameters together with the stress state in the vicinity of the corner point and the results [23] are defined. In what follows, they can be used similarly to the "trident" model developed in [10] and [17] to calculate the parameters of the zone of brittle prefracture which propagates at the following stage along the bisector of the break angle. The purpose of the present work is to determine the rate of energy release to be employed in the energetic criterion of the crack initiation.

Authors honor the memory of Prof. L. A. Kipnis who has initiated the present work.

1. Problem Statement. Consider a plane strain problem for a piecewise homogeneous body with the broken interface of two dissimilar isotropic materials with the elastic moduli E_1, E_2 and Poisson's ratios ν_1, ν_2 joined by plastic adhesive. The body is under the action of external load and initiates the local field of stresses and strains in the vicinity of the corner point of the interface symmetric about the bisector of the break angle.

The corner point of the interface under symmetrical load represents a stress concentrator with the power singularity [4] which leads to initiation in its vicinity of a narrow plastic zone in adhesive material and a process zone in one of joined materials. The parameters of the small-scale plastic zone and the stress-strain state near the vertex of the break angle due to its presence are defined in [23]. With the local symmetry principle [3, 14] (the local intensity factor for the mode *2* at the end of the process zone $k_2 = 0$), the process zone is oriented along the bisector of the interface break angle.

The process zone initiates the crack in it which necessitates the determination of its parameters. To solve the problem, we model the process zone by the line of discontinuity of the normal displacement with the length d_i (i = 1 or 2 is the material number) on which the normal stress is equal to the tear resistance σ_i of the i-th material of the joint. To consider the plastic zone in adhesive material in accordance with [23], we present it as the line of discontinuity of the tangential displacement of the length l on which the tangential stress is equal to the yield point of the material τ_s. The design schema of the problem on determining the parameters of the process zone is shown in Fig. 1 for the case when the zone arises in the first material. The alternative process

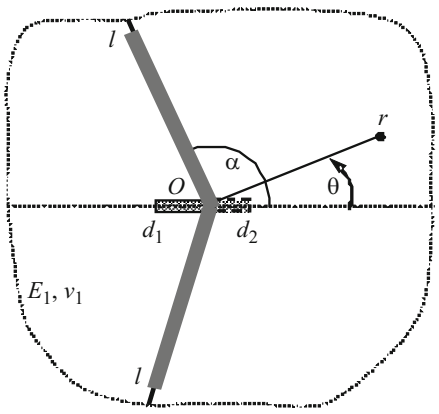


Fig. 1

zone in second material with the dashed boundary is shown by black color. The tearing normal stresses on the bisector of the break angle are the condition according to which the process zone should be formed in the *i*-th material.

It is assumed that the length of the plastic zone is strongly less than the sizes of piecewise homogeneous body but is considerably more than the length of the process zone. Considering the above-mentioned, we arrive to a plane static symmetrical elasticity problem for a piecewise homogeneous isotropic plane with the interface formed by the angle sides. The angle contains the half-finite lines of discontinuity of the tangential displacement emerging from the corner point and located on the interface as well as the cut of the finite length d_i emerging from the corner point along the bisector of the break angle in the *i*-th material ($d_i << l$, $l \to \infty$, Fig. 1). Assume that the break angle of the interface in second material is equal to 2α.

To formulate conditions at infinity, we use the method of joining of asymptotic solutions [27] according to which the desired solution at the distances $d_i << r << l$ should pass into the known asymptotics of the solution of the identical problem [23] without the process zone but with the finite-length plastic zones in adhesive material. Considering the symmetry, we restrict our consideration of the domain $0 \le \theta \le \pi$ and arrive at the elasticity problem with the boundary-value conditions (Fig. 1)

$$\theta = 0 \cup \pi, \quad \tau_{r\theta} = 0, \quad u_\theta = 0,$$

$$\theta = \alpha, \quad \langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0, \quad |\tau_{r\theta}| = \tau_s, \quad \langle u_\theta \rangle = 0, \tag{1}$$

$$\theta = \beta_i, \quad r < d_i, \quad \sigma_\theta = \sigma_i,$$

$$\theta = \beta_i, \quad r > d_i, \quad u_\theta = 0, \tag{2}$$

$$\theta = \beta_i, \quad r \to \infty, \quad \sigma_\theta = Q_i r^\lambda + o(1/r), \tag{3}$$

where $\langle f \rangle$ is the discontinuity of the value *f* when crossing the appropriate section; $\beta_1 = \pi$ and $\beta_2 = 0$ in the cases when the process zone appears either in the first or second material, respectively; Q_i and λ (index of the stress singularity near the break point, $\lambda \in (-1, 0)$) are the functions of the break angle and elastic characteristics of the materials defined in [23] from the solution of the problem on plastic zones in adhesive material. The tensile normal stress $\sigma_\theta(r, \beta_i) > 0$ should act on the section of the bisector where the appearance of the process zone is assumed, providing the tear strains necessary for the zone formation. In accordance with (3), this condition is equivalent to the inequality $Q_i > 0$. The equation for calculating the singularity index λ and functions Q_i is presented in Appendix A.

In approaching the end of the process zone, the asymptotics for stresses and displacements, being the asymptotically largest solution of the homogeneous elasticity problem for the homogeneous plane including the half-infinite straight line of discontinuity of the normal displacement, is realized. Particularly, the asymptotics are as follows

$$\theta = \beta_i, \quad r \to d_i + 0: \quad \sigma_\theta \sim \frac{k_i}{\sqrt{2\pi(r - d_i)}},$$